Desert Island Survival

Physics 101.

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“Give me a fulcrum and I will move the world” - shouted Archimedes, finding a perfect solution. Which is the best? A solution implemented with a perfect tool, or the Desert Island Survival Solution – one, executed with bare hands?

A student stuck at a Desert Island on her way back from the Spring Break. Being inspired by her teacher’s solving problems via reasoning from the conceptual principles all the way to the answer; she did come up with a result without the unavailable Internet, but had an error that led to an incorrect answer.

Another student based his answer on the information acquired from the Internet. Just a few steps led to the absolutely correct solution. Which work you would assign a higher grade?

The author will share some examples of providing unnecessary information as well as examples of how to find solutions from limited, but sufficient conceptual information [1,2].

Perfect tool versus bare hands
Mathematician

Physicist

Engineer

Volume of the blue rubber ball

\[ V = 2 \int_0^r \pi x^2 \, dy = 2 \pi \int_0^r (r^2 - y^2) \, dy = \]

\[= 2\pi \left[ r^2 y - \frac{y^3}{3} \right]_0^r = 2\pi \left( r^3 - \frac{r^3}{3} - \left( 0^3 - \frac{0^3}{3} \right) \right) = \]

\[= 2\pi \frac{2r^3}{3} = \frac{4\pi r^3}{3} \]
Brewing tea
A Mathematician and a Physicist

Case One

Case Two
Radius of the Earth

From the map you estimate the distance to the opposite side of the lake as d=6.1 km.

Why not Google for the Radius of the Earth?

Why not use what you learn from the previous example?

EXAMPLE 1-9 ESTIMATE Estimating the radius of Earth. Believe it or not, you can estimate the radius of the Earth without having to go into space (see the photograph on page 1). If you have ever been on the shore of a large lake, you may have noticed that you cannot see the beaches, piers, or rocks at water level across the lake on the opposite shore. The lake seems to bulge out between you and the opposite shore—a good clue that the Earth is round. Suppose you climb a stepladder and discover that when your eyes are 10 ft (3.0 m) above the water, you can just see the rocks at water level on the opposite shore. From a map, you estimate the distance to the opposite shore as d = 6.1 km. Use Fig. 1–14 with h = 3.0 m to estimate the radius R of the Earth.

APPROACH We use simple geometry, including the theorem of Pythagoras,

\[ c^2 = a^2 + b^2, \]

where \( c \) is the length of the hypotenuse of any right triangle, and \( a \) and \( b \) are the lengths of the other two sides.

SOLUTION For the right triangle of Fig. 1–14, the two sides are the radius of the Earth \( R \) and the distance \( d = 6.1 \text{ km} = 6100 \text{ m} \). The hypotenuse is approximately the length \( R + h \), where \( h = 3.0 \text{ m} \). By the Pythagorean theorem,

\[ R^2 + d^2 = (R + h)^2 \]
\[ R^2 + d^2 = R^2 + 2hR + h^2. \]

We solve algebraically for \( R \), after cancelling \( R^2 \) on both sides:

\[ R \approx \frac{d^2 - h^2}{2h} = \frac{(6100 \text{ m})^2 - (3.0 \text{ m})^2}{2 \times 6.0 \text{ m}} \]
\[ = 6.2 \times 10^6 \text{ m} \]
\[ = 6200 \text{ km}. \]

NOTE Precise measurements give 6380 km. But look at your achievement! With a few simple rough measurements and simple geometry, you made a good estimate of the Earth’s radius. You did not need to go out in space, nor did you need a very long measuring tape.
Height by triangulation

Another approach, this one made famous by Enrico Fermi (1901-54, Fig. 1-13), was to show his students how to estimate the number of pianos tuned in a city, say, Chicago or San Francisco. To get a rough order-of-magnitude estimate of the number of piano tuners today in San Francisco, a city of about 800,000 inhabitants, we can proceed by estimating the number of functioning pianos, how often each piano is tuned, and how many pianos each tuner can tune. To estimate the number of pianos in San Francisco, we note that certain of

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One dimensional motion with the constant acceleration

\[ a = \text{const} \Rightarrow \]

\[
x(t) = x_0 + V_0 t + \frac{1}{2} a t^2
\]

\[
v(t) = v_0 + at
\]

\[
x_0 = x(t) - V_0 t - \frac{1}{2} a t^2
\]

\[
v_0 = \frac{x(t) - x_0}{t} - \frac{1}{2} a t
\]

\[
v_0 = v(t) - at
\]

\[
a = \frac{v(t) - v_0}{t}
\]

\[
v_{0y} = \pm \sqrt{v_{0y}^2 - 2g \cdot y(t)}
\]

\[
t = \sqrt{\frac{2g}{(x-x_0)\tan \theta - (y-y_0)}}
\]
Atwood’s Machine

\[ T - m_1 g = m_1 a \]
\[ m_2 g - T = m_2 a \]

\[ T = m_1 a + m_1 g = m_1 (a + g) \]

\[ g (m_2 - m_1) = a (m_1 + m_2) \Rightarrow \]

\[ T = g \left( \frac{m_2 - m_1}{m_1 + m_2} \right) m_1 + m_1 \left( m_1 + m_2 \right) \]

\[ T = g \frac{2m_1 m_2}{m_1 + m_2} \]

\[ m_2 = m_1 \Rightarrow \]
\[ a = g \frac{0}{2m_1} = 0 \]
\[ T = g \frac{2m_1 m_2}{2m_1} = m_2 g \]

\[ m_2 ? m_1 \Rightarrow \]
\[ a = g \frac{m_2 - 0}{0 + m_2} = g \]
\[ T = g \frac{2 \times 0 \times m_2}{0 + m_2} = 0 \]

\[ m_2 = m_1 \Rightarrow \]
\[ a = g \frac{0 - m_1}{m_1 + 0} = -g \]
\[ T = g \frac{2m_1 \times 0}{m_1 + 0} = 0 \]

\[ 2.9 \]
Applying what you learn in one field to another one

Learn how to see similarity in things looking absolutely different

Learn how to see difference in things looking absolutely same
**HARMONIC OSCILLATOR**

**Hook’s Law:**

\[
\begin{align*}
F &= -kx \\
F &= ma
\end{align*}
\]

\[
\Rightarrow a = -\frac{k}{m} x \quad \Leftrightarrow \quad \Delta \left( \frac{\Delta x}{\Delta t} \right) = -\frac{k}{m} x
\]

\[
\frac{k}{m} = \omega^2 \quad \Rightarrow \quad \Delta \left( \frac{\Delta x}{\Delta t} \right) = -\omega^2 x
\]

\[
x = A \cos (\omega t + \phi)
\]

\[
T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{m}{k}}
\]

\[
f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{k}{m}}
\]

**natural frequency**
SIMPLE PENDULUM

\[ F = -mg \sin \theta \]

small \( \theta \) \( \Rightarrow \) \( F \approx -mg \theta \)

\[ \theta = \frac{s}{L} \]

\[ F \approx -mg \frac{s}{L} = -\frac{mg}{L} s = -kx \]

where \( k = \frac{mg}{L} \)

\[ T = 2\pi \sqrt{\frac{m}{k}} = 2\pi \sqrt{\frac{m}{mg/L}} \Rightarrow \]

\[ T = 2\pi \sqrt{\frac{L}{g}} \Rightarrow \]

\[ f = \frac{1}{T} = \frac{1}{2\pi} \sqrt{\frac{g}{L}} \text{ natural frequency} \]

\[ = 1 \text{ HZ} = \frac{1}{s} = 1 s^{-1} \]

Hertz
Jigsaw puzzle mentality