# SIMPLE HARMONIC MOTION

## GOALS

- To investigate the qualitative response of a physical system subject to a variable force.
- To investigate the quantitative relationship between the characteristics of simple harmonic motion (amplitude, period, phase) and the physical characteristics of the systems while undergoing simple harmonic motion (length and mass of pendulum, elastic constant, etc.)
- To explore the relationship between displacement, velocity, acceleration and force.
- To explore the relationship between the potential and kinetic energy for simple harmonic motion (SHM).
- To introduce students to computer-based data acquisition techniques and to demonstrate how a computer can be used to actively interface in experiments.

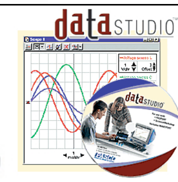
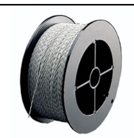
## OBJECTIVES

After successfully completing this lab, the students should be able to:

- Define and build a simple pendulum and a simple harmonic oscillator.
- Use Newton's second law to derive the relationship between acceleration and displacement.
- Define the angular frequency, period of oscillation, frequency and the amplitude of oscillation for a simple pendulum and for a simple harmonic oscillator.
- Define elastic potential energy and kinetic energy.
- Know how to connect the motion sensor and force sensor to the data acquisition interface.
- Know how to connect the data acquisition interface to a computer.
- Know how to start a new experiment using the DataStudio software.
- Prepare a written report presenting the measurements.

## EQUIPMENT

| Description | Photo |
|---|---|
| **Force sensor**<br>- ± 50 N range |  |
| **Springs** |  |
| **Motion Sensor** uses ultrasonic pulse ranging technology to focus on the target<br>- Reports position, velocity and acceleration<br>- 0.15 to 8 m range and 1.0 mm resolution |  |

| | |
|---|---|
| **PASCO 750 Interface** is the measurement center<br>- 4 digital and 3 analog ports<br>- USB connection to computer |  |
| **Laptop computer with DataStudio software**<br>- Automatically and manually input or import data from sensors or other sources.<br>- Analyze, graph, print, and export data collected from sensors. |   |
| **Masses**<br>- Slotted Mass Set (1 g resolution)<br>- Slotted Mass Hanger<br>- Hooked Mass Set |    |
| **String**<br>- Tough, stretch-resistant and able to withstand at least 10 N of force (equivalent to about 1 kg). |  |

**THEORY**

Periodic motion is a motion that repeats itself. The repetition of motion can be easily visualized by plotting the position, velocity, and/or acceleration. The time it takes for the system to go through one cycle is called the **period**, *T*, and is measured in seconds. The number of cycles in each second is called the **frequency**, *f*, and is measured in cycles-per-second, or Hertz (Hz). The period and frequency are related through $f = 1/T$. The "center" of the motion is called the **equilibrium point**. The maximum displacement from the equilibrium position is called the **amplitude**. All the derivations in this laboratory assume that air friction and any other dissipative forces are negligible.

**A simple pendulum** consists of a mass *m* and an inextensible string of length *l* (see Figure 1a). A simple pendulum has a stable equilibrium, which is the vertical direction. Any displacement of the pendulum from equilibrium point produces small oscillations of equal amplitude on both sides of the equilibrium position.

A force sensor measures the tension in the string as it swings left and right around equilibrium. The component of the weight of the bob that is tangent to the trajectory forces the bob back to equilibrium. The normal (perpendicular to the trajectory) component of the weight determines the tension in the string. Newton's second law determines both tangential and the normal accelerations of the pendulum:

$$\vec{W} + \vec{N} = m\vec{a} \Rightarrow \begin{cases} \sum F_x = -mg\sin\phi = ma_x = m(l\alpha) \\ \sum F_y = N - mg\cos\phi = ma_y = 0 \end{cases} \Leftrightarrow \begin{cases} \alpha = -\dfrac{g}{l}\sin\phi = -\omega_0^2\sin\phi \\ N = mg\cos\phi \end{cases} \quad (1)$$

where W is the weight of the bob, N is the tension force, $\alpha$ is the tangential angular acceleration corresponding to the angular displacement $\phi$, $\omega_0 = \sqrt{g/l}$ is called the angular frequency and its units are radians/second. The period of oscillation is always related to the angular frequency through $\omega_0 = 2\pi/T$, which leads, in the case of the simple pendulum, to

$$T = 2\pi\sqrt{l/g}\,. \quad (2)$$

For small amplitude oscillations ($\phi < 5^o$), sin $\phi \approx \phi$, and therefore, eq. (1a) states the acceleration forcing the object to return to its equilibrium position is proportional to angular displacement, which be shown mathematically that it leads to harmonic oscillations of a constant period. Eq. (1b) simply determines the magnitude of the tension force.

**A simple harmonic oscillator** consists of a mass *m* attached to a spring of elastic constant *k* (Figure 1b and 1c).

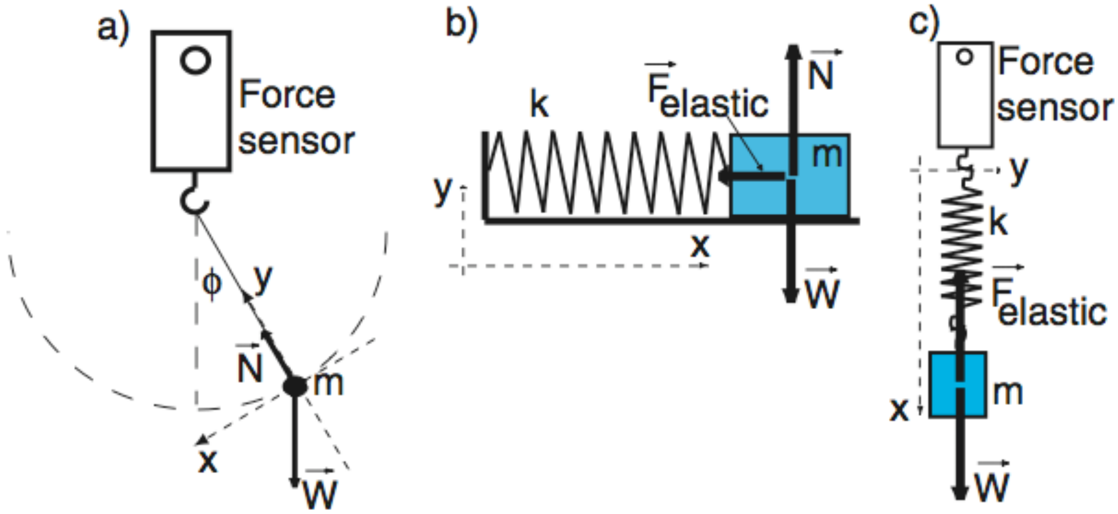



Figure 1. Simple pendulum (a) and simple harmonic oscillator on a horizontal frictionless surface (a) or oscillating vertically (b).

A small displacement *x* from equilibrium (unstretched spring) position determines a force inside the spring that is proportional to the displacements according to Hook's law:

$$F_{elastic} = -kx. \quad (3)$$

Therefore, Newton's second law for small amplitude oscillations of the simple harmonic oscillator shown in Figure 1b is

$$\vec{F}_{elastic} + \vec{W} + \vec{N} = m\vec{a} \Rightarrow \begin{cases} \sum F_x = -kx = ma_x \\ \sum F_y = N - mg = 0 \end{cases} \Leftrightarrow \begin{cases} a_x = -\dfrac{k}{m}x = -\omega_0^2 x \\ N = mg \end{cases} \quad (4)$$

where $\omega_0 = \sqrt{k/m}$ is the angular frequency. Again, using the relationship $\omega_0 = 2\pi/T$, the period of small amplitude oscillations in the case of the simple harmonic oscillator becomes

$$T = 2\pi\sqrt{m/k}\,. \quad (5)$$

As in the previous section, the acceleration along the horizontal direction $a_x$ is proportional to the linear displacement, and, therefore, the mass *m* describes a harmonic oscillation.

The fact that the system lies on a frictionless horizontal surface (Figure 1b) or hangs vertically (Figure 1c) is irrelevant for the period of oscillation.

The **elastic potential energy** stored by a spring compressed/stretched by *x* units is $U_{elastic} = ½\, kx^2$, where *k* is the elastic constant of the spring.

The **kinetic energy** of an object of mass *m* is $KE = ½\, mv^2$, where *v* is the velocity of the object.

The sum of potential and kinetic energy is called **mechanical energy**.

## A SIMPLE PENDULUM

**Experimental setup**

1. Cut a string, hang a mass at one end and tie the other end to the force sensor hook (see Figure 1a).
2. Connect the interface to the computer, turn on the interface, and turn on the computer.
3. Connect one force sensor to one of the three analog channels on the interface.
4. Run DataStudio on the computer and open the file named "simple_pendulum.ds". The file corresponds to the physical setup and collects the values of the tension over time.

**Experiment 1.** How does the initial angular position affect the period of oscillation?

***Prediction***

*Suppose the same simple pendulum starts repeatedly with different initial angular positions, what would you expect the period of oscillation to be in each case? Explain your reasoning on the Data/Question sheet.*

**Experimental procedure**

1. Record the length of the string and the mass of the bob on the Data/Question sheet.
2. Move the bob away from equilibrium position with a small angle determined with a protractor and click the “Start” button in your DataStudio file to record the tension inside the string. Click the “Stop” button after you captured about three or four complete oscillations such that you have enough cycles to estimate the period of oscillation and, at the same time, the amplitude of oscillation remains almost constant.
3. When you are satisfied with your data, use the force versus time graph to estimate the average period of oscillation. Record your values in Table 1 on the Data/Question sheet.

In order to determine the period of oscillation, click on the “Smart tool” icon and move the cross hair on your graph over the successive points of maximum amplitude of the oscillations. Determine the time interval between successive maxima and take the average over multiple (three or four) cycles. Record the average period in Table 1 on the Data/Question sheet.

**Experiment 2.** How does mass affect the period of oscillation?

***Prediction***

*Suppose the same simple pendulum of constant length starts repeatedly with the same initial angular displacement but different masses, what would you expect the period of oscillation to be in each case? Explain your reasoning on the Data/Question sheet.*

**Experimental procedure**

1. Record the length of the string, the angular amplitude ($\phi_{max} < 5^o$) and the mass of the bob on Table 2 of the Data/Question sheet.
2. Move the bob away from equilibrium position with a small angle ($\phi_{max} < 5^o$) and click the “Start” button on DataStudio to begin recording the tension inside the string.

Click the "Stop" button after capturing about three or four complete oscillations such that enough cycles are available for a reasonable estimate of the period of oscillation and, at the same time, the amplitude of oscillation remains almost constant.

3. When you are satisfied with your data, use the force versus time graph to estimate the period of oscillation and record it as the experimental period $T_{exp}$ in Table 2 on the Data/Question sheet.
4. Use formula (2) to compute the theoretical period of oscillation $T_{th}$ and record it in Table 2 on the Data/Question sheet. In addition, calculate the percent difference

$$\% \text{ difference} = \frac{|T_{\exp} - T_{th}|}{T_{th}} 100\% .$$

**Experiment 3.** How does the pendulum length affect the period of oscillation?

***Prediction***

*Suppose the same simple pendulum of constant mass starts repeatedly with the same initial angular displacement but having different lengths each time, what would you expect the period of oscillation to be in each case? Explain your reasoning on the Data/Question sheet.*

**Experimental procedure**

1. Record the mass of the bob, the angular amplitude ($\phi_{max} < 5^o$), and the length of the string on Table 3 of the Data/Question sheet.
2. Move the bob away from the equilibrium position with a small angle ($\phi_{max} < 5^o$) and click the "Start" button to begin recording the tension inside the string. Click the "Stop" button after you captured about three or four complete oscillations.
3. When you are satisfied with your data, use the force versus time graph to estimate the period of oscillation and record it as the experimental period $T_{exp}$ in Table 3 on the Data/Question sheet.
4. Use formula (2) to estimate the theoretical period of oscillation $T_{th}$ and record it in Table 2 on the Data/Question sheet. In addition, calculate the percent difference

$$\% \text{ difference} = \frac{|T_{\exp} - T_{th}|}{T_{th}} 100\% .$$

## B SIMPLE HARMONIC OSCILLATOR

**Experimental setup**

1. Hang the force sensor vertically from a support and attach a spring to its hook. Attach a slotted mass hanger to the other end of the spring.
2. Place the motion detector facing up directly below the spring and cover it always with the provided protective cage. Push the mass straight up about 5-10 cm and let it oscillate. Adjust the height of the support so that the mass comes no closer than 20 cm to the detector. If necessary, tape a small card to the bottom of the mass. Use the protective wire cage to protect the motion detector.
3. Connect the interface to the computer, turn on the interface, and turn on the computer.
4. Connect one force sensor to one of the three analog (DIN) channels on the interface.
5. Run DataStudio on the computer and open the file named "simple_harmonic_oscillator.ds". The file corresponds to the physical setup and collects the values of the tension in the spring and the position/velocity/acceleration of the hanging mass.

**Experiment 4.** Measure the elastic constant of the spring

***Prediction***

*Suppose different masses are hung on the same spring at different times and the elastic constant of the spring is measured each time. What would you expect the elastic constant to be in each case? Explain your reasoning on the Data/Question sheet.*

**Experimental procedure**

1. Start the mass oscillating by gently pushing it downward (make sure that the mass never comes closer than 20 cm to the motion detector). When the motion is smooth, click the "Start" button to begin collecting data. Adjust the vertical axes of the graphs so that they are easier to read. Sketch or save a copy of your graph on the Data/Question sheet.
2. One of the graphs is set to plot the tension versus the displacement from equilibrium. You should get a straight line. Click on the fitting tool and select the linear fit option. Inside the annotated box that contains the fit equation you can read the slope, which is the elastic constant of the spring.
3. Add more slotted masses such that the total hanging mass is 250 g and repeat steps 1 and 2.
4. Add more slotted masses such that the total hanging mass is 500 g and repeat steps 1 and 2.
5. Make a conclusion about whether the mass affects the spring constant.

**Experiment 5.** Simple harmonic oscillator of variable mass

***Prediction***

*Suppose different masses are hung on the same spring at different times. What would you expect* *the period of oscillation to be in each case? Explain your reasoning on the Data/Question sheet.*

**Experimental procedure**

1. Start the mass oscillating by gently pushing it downward (make sure that the mass never comes closer than 20 cm to the motion detector). When the motion is smooth, click the "Start" button to begin collecting data. Adjust the vertical axes of the graphs so that they are easier to read. Sketch or save a copy of your graph on the Data/Question sheet.
2. One of the graphs is set to plot the tension versus time. Use it to estimate the period of oscillations and record the results in Table 3 on the Data/Question sheet.
3. Add more slotted masses such that the total hanging mass is 250 g and repeat steps 1 and 2.
4. Add more slotted masses such that the total hanging mass is 500 g and repeat steps 1 and 2.
5. Make a conclusion about how the mass affects the period of oscillation.

**Experiment 6.** Simple harmonic oscillator of constant mass and variable elastic constant.

***Prediction***
*Suppose the same mass is hung on different springs at different times. What would you expect the period of oscillation to be in each case? Explain your reasoning on the Data/Question sheet.*

**Experimental procedure**

1. For each spring, start the mass oscillating by gently pushing it downward. When the motion is smooth, start collecting data. Adjust the vertical axes of the graphs so that the measurements are easier to read.
2. The first graph of interest is the plot of force versus distance. You should get a straight line for each spring. Click on the fitting tool and select the linear fit option. Inside annotated box that contains the fit equation you can read the slope, which is the elastic constant of the spring. Write down the value of the elastic constant in Table 6 on the Data/Question sheet.
3. The second graph of interest is force versus time. Estimate the period of oscillation from such a graph and write it down in Table 6 on the Data/Question sheet.
4. Repeat steps 1 to 4 for at least three different springs.
5. Make a conclusion about how the elastic constant affects the period of oscillation.

# SIMPLE HARMONIC MOTION
# DATA/QUESTION SHEET

## SIMPLE PENDULUM

**Experiment 1.** How does the initial angular position affect the period of oscillation? Simple pendulum of constant length and mass and variable initial angular displacement.

***Prediction***
*Suppose the same simple pendulum starts repeatedly with different initial angular position, what would you expect the period of oscillation to be in each case? Explain your reasoning on the Data/Question sheet.*

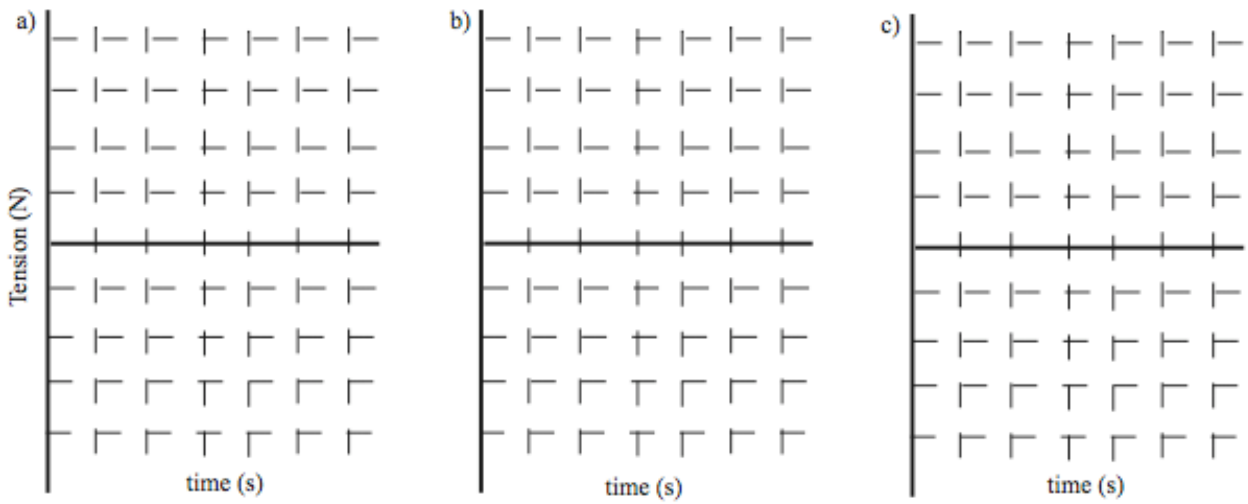



Figure 2. Sketch (or export from DataStudio and insert here) the graph of the tension inside the string versus time for (a) small, (b) moderate, and (c) large angular amplitude.

Table 1. Simple pendulum of length $l$ = ______ m and mass $m$ = ______ kg started with different initial angular displacements $\phi_{max}$. The purpose of the experiment is to establish the relationship between the period of oscillation T and the amplitude of oscillation $\phi_{max}$.

| $\phi_{max}$ (degrees) | T (s) |
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Plot the period of oscillation versus the angular displacement $\phi_{max}$. Does it appear that the total period is independent of $\phi_{max}$? For what range of $\phi_{max}$?

**Experiment 2.** How does mass affect the period of oscillation? Simple pendulum of constant length and initial angular displacement with variable mass.

***Prediction***

*Suppose the same simple pendulum of constant length starts repeatedly with the same initial angular displacement but different masses, what would you expect the period of oscillation to be in each case? Explain your reasoning on the Data/Question sheet.*

Table 2. Simple pendulum of length *l* = ______ m and initial angular displacement $\phi_{max}$ = ______ degrees. The purpose of the experiment is to establish the relationship between the period of oscillation T and the mass of the pendulum.

| m (kg) | $T_{exp}$ (s) | $T_{th} = 2\pi\sqrt{l/g}$ (s) | % difference |
|---|---|---|---|
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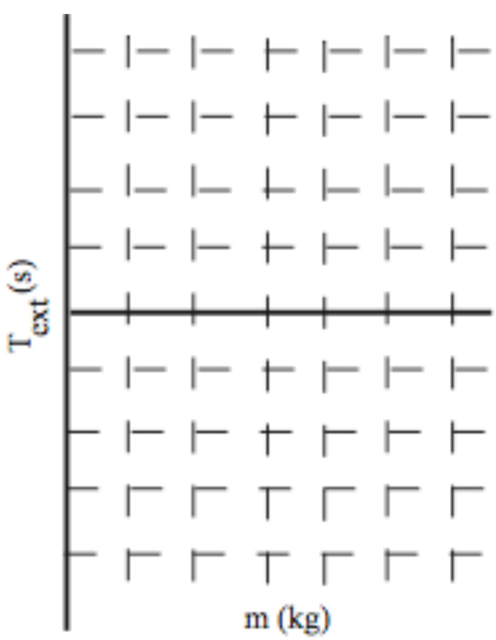


Figure 3. Plot the experimental period of oscillation $T_{exp}$ versus the hanging mass.

Does it appear that the experimental period of oscillation depends on the hanging mass?

Does it appear that the experimental period of oscillation $T_{exp}$ is accurately described by the theoretical formula (2)?

**Experiment 3.** How does the pendulum's length affect the period of oscillation? Simple pendulum of constant mass and initial angular displacement with variable length.

***Prediction***

*Suppose the same simple pendulum of constant mass starts repeatedly with the same initial angular displacement but different lengths, what would you expect the period of oscillation to be in each case? Explain your reasoning on the Data/Question sheet.*

Table 3. Simple pendulum of mass $m$ = ______ kg and initial angular displacement $\phi_{max}$ = ______ degrees. The purpose of the experiment is to establish the relationship between the period of oscillation T and the length $l$ of the pendulum.

| l (m) | $T_{exp}$ (s) | $T_{th} = 2\pi\sqrt{l/g}$ (s) | % difference |
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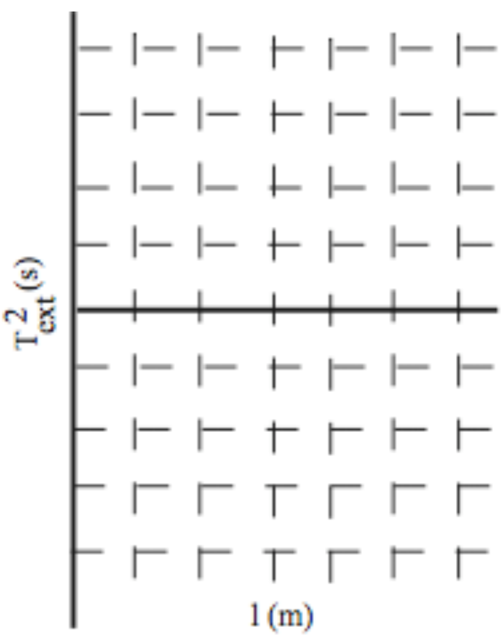


Figure 4. Plot the square of the experimental period of oscillation $(T_{exp})^2$ versus the length of the pendulum.

What kind of relationship (linear, quadratic, etc.) seems to exist between $(T_{exp})^2$ and $l$?

Does it appear that the experimental period of oscillation $T_{exp}$ is accurately described by the theoretical formula (2)?

**SIMPLE HARMONIC OSCILLATOR**

**Experiment 4.** Measure the elastic constant of the spring.

***Prediction***
*Suppose different masses are hung on the same spring at different times and the elastic constant of the spring is measures each time. What would you expect the elastic constant to be in each case? Explain your reasoning.*

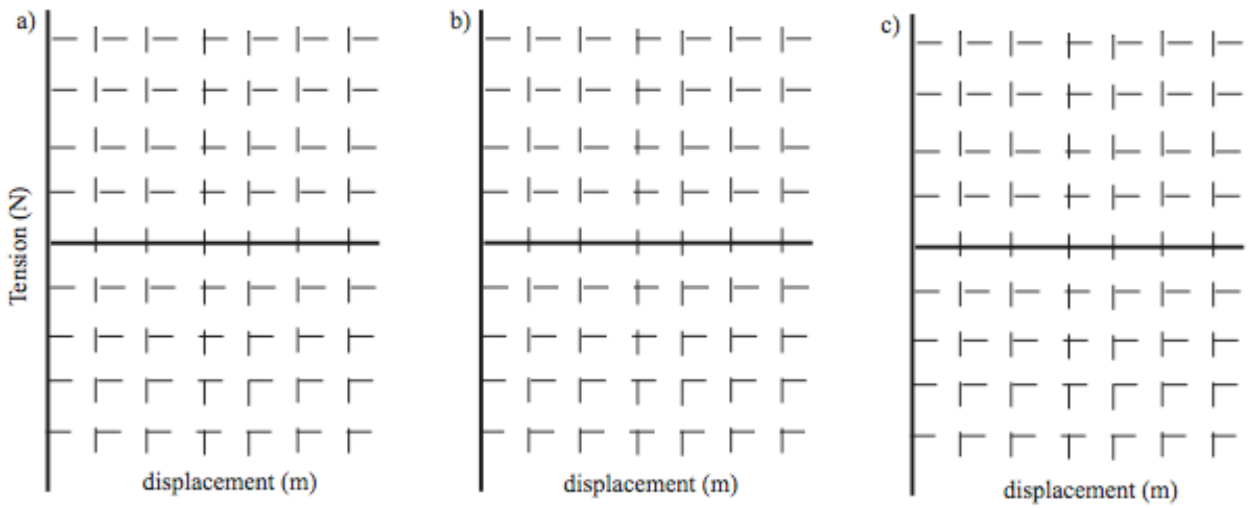


Figure 5. Force versus displacement from equilibrium position for a simple harmonic oscillator. The slope of the linear interpolation of experimental data is the elastic constant of the spring for small (a), moderate (b) and large (c) hanging masses.

Table 4. Simple harmonic oscillator with variable mass.

| m (kg) | k (N/s) |
|---|---|
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Does it appear that the elastic constant depends on the hanging mass?

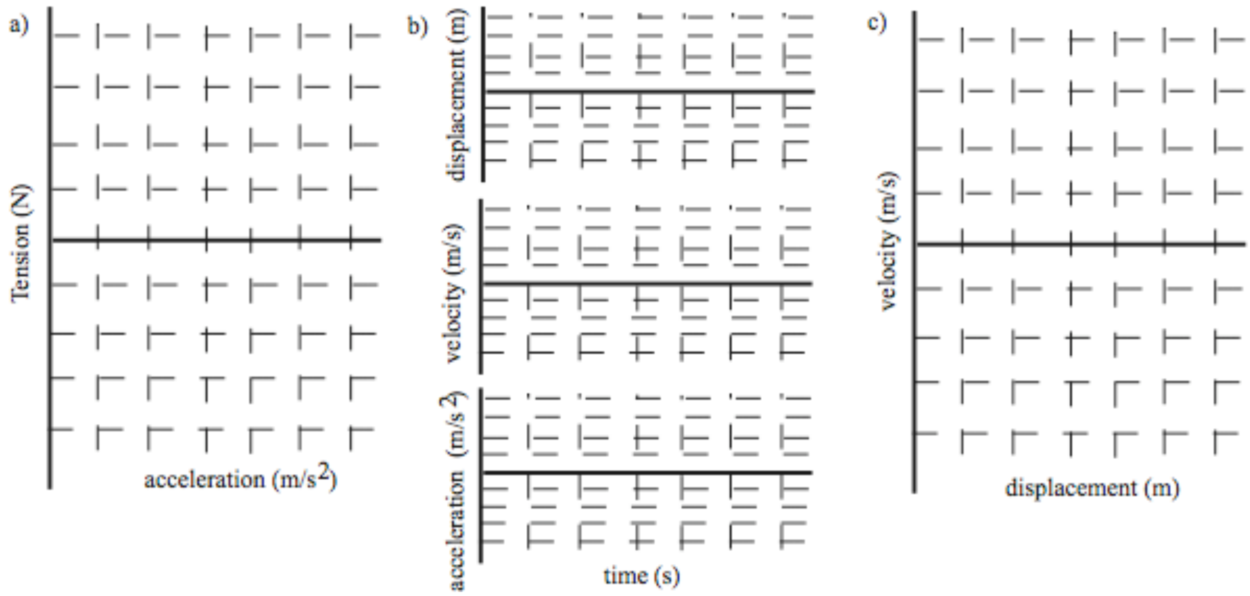


Figure 6. Sketch (or export from DataStudio and insert here) on the same panel the graphs of the tension and acceleration versus time for (a) displacement, velocity and acceleration versus time, (b) velocity versus displacement (c).

Based on the graph of force and acceleration versus time (Figure 6a), does there appear to be a relationship between these two quantities? Explain.

Based on the graph of displacement, velocity, and acceleration versus time (Figure 6b), does there appear to be any relationship between the phases of these quantities? Is the graph of displacement versus time similar (*i.e.* the same period of oscillation and the same phase) to either of the other two graphs? How many independent variables (displacement, velocity, acceleration) do you actually need to describe a simple harmonic motion? Explain.

The graph of velocity versus displacement (Figure 6c) should be a closed curve, which is called a phase space trajectory. However, if you run the experiment for a long time, then the amplitude of oscillation decreases over time. Describe the phase space trajectory for a long run (tens of cycles)? What will happen to the phase space trajectory for a very long (“infinity”) running time?

Based on the graph of elastic potential energy and kinetic energy (Figure 7), does there appear to be a relationship between these quantities? Are they oscillating in phase or out of phase? Are they oscillating with the same period? Explain.

Is mechanical energy (Figure 7c) constant? What happens to mechanical energy if you run the experiment for a very long time? Why is mechanical energy decreasing? Where is the energy "disappearing"?

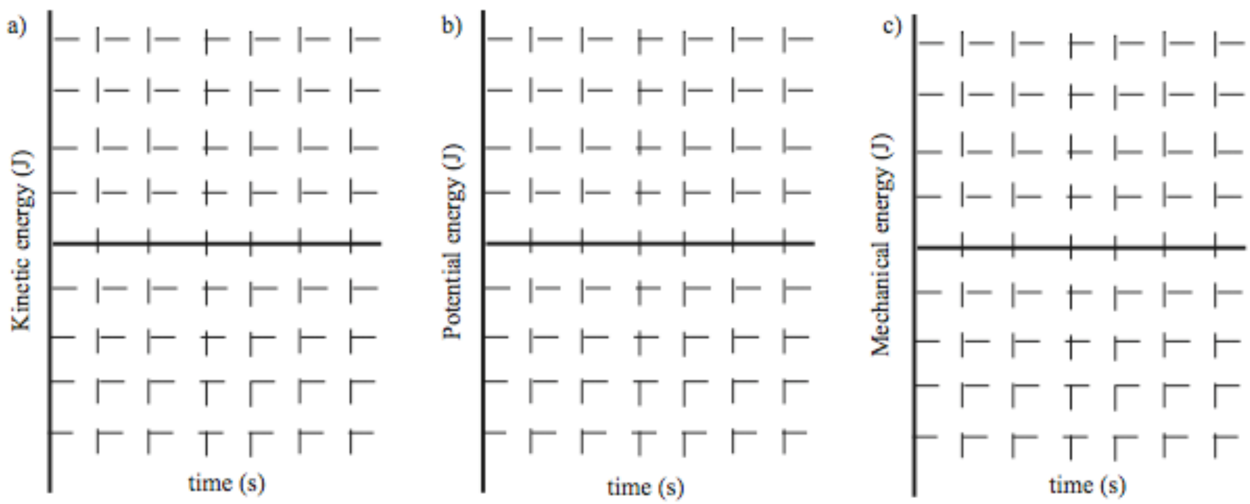


Figure 7. Sketch (or export from DataStudio and insert here) on the same panel the graphs of elastic potential energy and kinetic energy (a), and the mechanical energy versus time (b).

**Experiment 5.** Simple harmonic oscillator of variable mass

***Prediction***

*Suppose different masses are hung on the same spring at different times. What would you expect the period of oscillation to be in each case? Explain your reasoning on the Data/Question sheet.*

Table 5. Simple harmonic oscillator with variable mass. A spring of elastic constant k = _______N/m oscillates around equilibrium.

| m (kg) | $T_{exp}$ (s) | $T_{th} = 2\pi\sqrt{m/k}$ | % difference |
|---|---|---|---|
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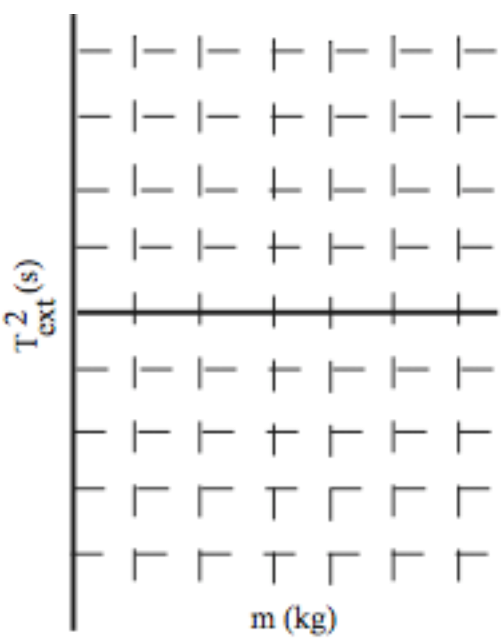


Figure 8. Plot the experimental period of oscillation $(T_{exp})^2$ versus the hanging mass.

Does it appear that the experimental period of oscillation depends on the hanging mass?

Does it appear that the experimental period of oscillation $T_{exp}$ is accurately described by the theoretical formula (5)?

**Experiment 6.** Simple harmonic oscillator of constant mass and variable elastic constants.

***Prediction***

*Suppose the same mass is hung on different springs at different times. What would you expect the period of oscillation to be in each case? Explain your reasoning on the Data/Question sheet.*

Table 6. Simple harmonic oscillator with the same mass m = _______ kg and variable elastic constants.

| k (N/m) | $T_{exp}$ (s) | $T_{th} = 2\pi\sqrt{m/k}$ | % difference |
|---|---|---|---|
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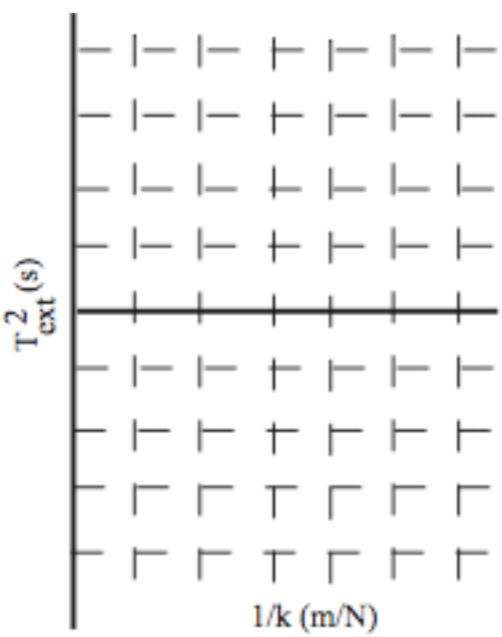


Figure 9. Plot the experimental period of oscillation $(T_{exp})^2$ versus the inverse of the elastic constant 1/k.

Does it appear that the experimental period of oscillation depends on the hanging mass?

Does it appear that the experimental period of oscillation $T_{exp}$ is accurately described by the theoretical formula (5)?