MOMENTUM AND COLLISIONS

GOALS

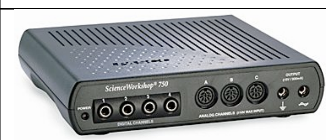
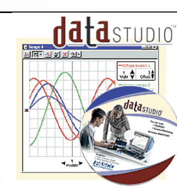
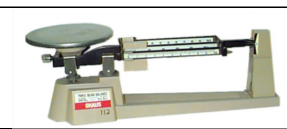
- To investigate the principle of conservation of linear momentum using one-dimensional elastic and inelastic collisions.
- To introduce students to computer-interfaced data acquisition and data analysis techniques.

OBJECTIVES

After successfully completing this lab, the students should be able to:

- Define linear momentum, impulse and kinetic energy, and the conservation of momentum.
- Explain the relationship between impulse and momentum.
- Know how to connect motion sensors, data acquisition interfaces and computers.
- Know how to start a new experiment using the DataStudio software.
- Complete and return at the end of class period the data tables and graphs summarizing all measurements and calculations regarding *Momentum and Collision* experiments.

EQUIPMENT

Description	Photo
Dynamic carts (2) - 250 gram mass (2)	
Dynamics System aluminum track - Adjustable endstop, rod clamp, adjustable feet	
Motion Sensor uses ultrasonic pulse ranging technology to focus on the target - Reports position, velocity and acceleration - 0.15 to 8 m range and 1.0 mm resolution	
PASCO 750 Interface is the measurement center - 4 digital and 3 analog ports - USB connection to computer	
Laptop computer with the DataStudio software - Automatically and manually input or import data from sensors or other sources. - Analyze, graph, print, and export data collected from sensors.	 
Triple Beam Balance - Reading error of 0.05 gram	

THEORY

Imagine a soccer player hitting a ball that was initially at rest. The player exerts a force on the ball for a short duration. We say that an impulse is applied to the object. The impulse is the product of the average net force exerted on an object and the time interval over which this force is applied: . As a result of the brief force applied by the player, the ball will gain speed. The linear momentum of an object of mass *m* is the product of the mass of the object and its velocity . The linear momentum is a vector and its direction is the same as the direction of the velocity. It was found that the change in linear momentum of an object is equal to the applied impulse:

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where p_i and p_f represent the initial and, respectively, final linear momentum along the direction of the average force . The above expression is known as the *Impulse-Momentum Theorem*.

For the simple case of collision between two objects, the impulse-momentum theorem for each object is

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In each of the above equations, the force has been broken into two parts: 1) the force exerted on one of the objects caused by the other object (F_{12} or F_{21}), and 2) an external term determined by other interactions except collision. According to Newton's third law, $F_{12} = -F_{21}$. Therefore, by adding the two equations above it results

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where is the total external force acting on the system formed by the two colliding masses,

and (1)

are the total final and, respectively, initial linear momenta. For one-dimensional motion, (1) becomes:

$$p_f = p_{1,f} + p_{2,f} = m_1v_{1,f} + m_2v_{2,f} \quad \text{and} \quad p_i = p_{1,i} + p_{2,i} = m_1v_{1,i} + m_2v_{2,i}. \qquad (2)$$

If the average external force is zero, then the final total linear momentum of the system is equal to the initial total linear momentum, which is referred to as *conservation of momentum*. The percent error on linear momentum estimates how close p_i and p_f are:

. (3)

Another important quantity that depends on the mass and velocity of the object is the kinetic energy, which is a scalar defined by . The total kinetic energy of a system of two objects is:

$$KE_f = ½m_1v_{1,f}^2 + ½m_2v_{2,f}^2 \text{ and } KE_i = ½m_1v_{1,i}^2 + ½m_2v_{2,i}^2. \qquad (4)$$

Similarly, the percent error on kinetic energy estimates how close KE_i and KE_f are:

. (5)

PROCEDURE - TOTALLY INELASTIC COLLISIONS

Definition: If two objects stick together after collision, then their collision is called *totally inelastic*.

Experimental setup for totally inelastic collisions

1. Level the track using the leveling screws on the track's feet (Fig. 1). A cart placed on a leveled track should not move by itself. Place a cart at each end of the track. Let the cart on the left be 'cart 1' and the cart on the right be 'cart 2'.
2. Use the carts with the Velcro sides toward each other so the carts will stick together after collision (totally inelastic collision).
3. Connect the interface to the computer, turn on the interface, and turn on the computer.
4. Connect one motion sensor into digital channels 1 and 2 on the interface. Connect the yellow plug into channel 1 and the other plug into channel 2.
5. Run DataStudio on the computer and open the file called "inellastic_collision.ds". The file corresponds to the physical setup shown in Fig. 1, and is responsible for collecting and displaying the values of the velocity of the left cart (cart 1) over time.

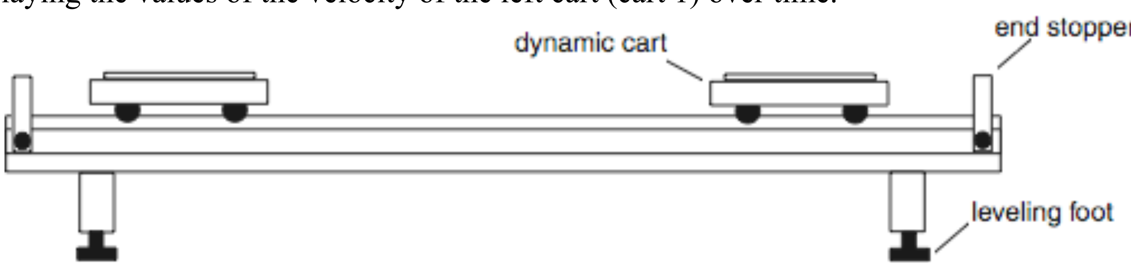

Figure 1. Totally inelastic collision setup: 2.2-m track with leveling feet, end stoppers, and two dynamic carts.

Experiment 1. One-dimensional totally inelastic collision with one cart moving and one stationary (Fig. 2)

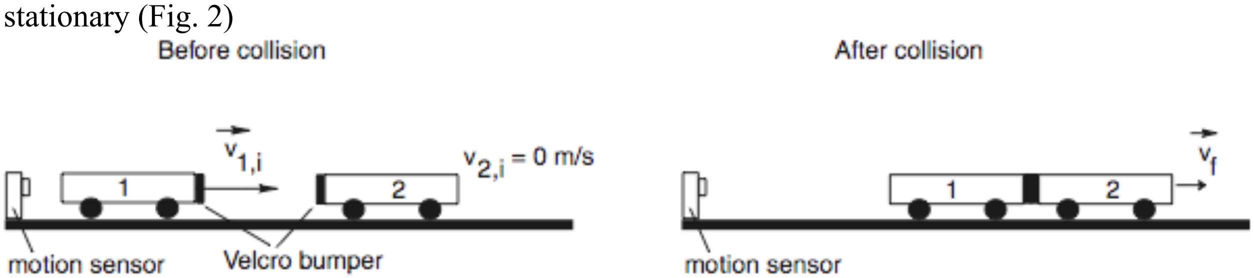

Figure 2. Experimental setup for one-dimensional totally inelastic collision with one cart moving and one stationary.

Prediction

Suppose two carts of masses m_1 and, respectively, m_2 collide inelastically. Knowing that before collision the first cart was in motion and the second at rest (Fig. 2), what would you expect the final velocity of the combined mass to be after collision? Explain your reasoning and show all your calculations.

Perform the following three experiments in order to check your prediction:

A. Equal mass carts.

Use the triple beam balance to find the mass of each cart and record the values in Table 1. Record the velocities obtained from steps 1 and 2 below in section A of Table 1.

1. With cart 2 stationary ($v_{2,i}$ = 0 m/s), give cart 1 a quick push and collect velocity data as it coast forward (the positive direction is always considered away from the motion detector). Stop recording soon after the two carts collide and before they hit the right end stopper. Practice until you obtain a graph of velocity versus time that has a distinct region of approximately constant velocity both before and after the collision point.
2. When you are satisfied with your data, use the velocity versus time graph to estimate the velocity of the first cart before collision ($v_{1,i}$) and the velocity of the two carts moving together after collision (v_f). Record the values in section A of Table 1 and carefully draw the velocity graph for cart 1 in the appropriate panel of Fig. 3. It might be helpful to expand the graph recorded with DataStudio to see just the area that you are interested in. To select a small portion of data from your DataStudio graph, click and hold the right button of the mouse while you move over the graph. Once you selected a region of interest with an almost constant velocity, use the statistical tool by clicking of the symbol Σ in the main menu. As a result, the mean value of the selected data will be displayed. Write the mean velocity before and after the collision point in section A of Table 1.

B. Heavy moving cart hitting the lighter stationary cart.

Place a mass bar in the bed of the first cart such that the heavy cart hits the lighter cart at rest and repeat the steps 1 and 2 of Experiment 1A above. Repeat the experiment with two and three bar weights on the moving cart. Record the values in section B of Table 1. Don't forget which cart is which!

C. Light moving cart hitting the heavier stationary cart.

Place a mass bar in the bed of the second (stationary) cart such that the lighter cart hits the heavier cart at rest and repeat steps 1 and 2 of Experiment 1A above. Repeat the experiment with two bar weights on the stationary cart. Record the values in section C of Table 1. Do not forget which cart is which!

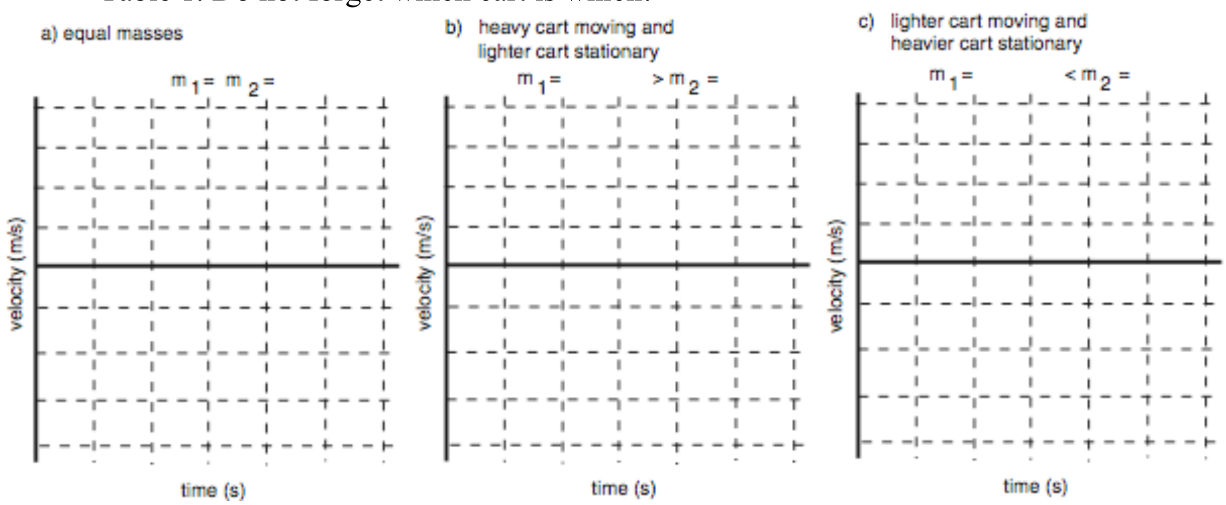

Figure 3. Sketch of the velocity versus time graph for one-dimensional inelastic collision with (a) equal masses, (b) a heavy moving cart hitting a lighter stationary cart, and (c) a light moving cart hits a heavier stationary cart. Mark on the graph the region over which the collision occurs.

Table 1. Momentum in one-dimensional totally inelastic collisions with one moving and one stationary cart.

Exp. 1	m_1 (kg)	m_2 (kg)	Before collision	After collision	$p_i = m_1 v_{1,i}$ (kg m/s)	$p_f=(m_1+m_2)v_f$ (kg m/s)	err_P (%)
			$v_{1,i}$ (m/s)	v_f (m/s)			
A							
B							
C							

Based on Table 1, does it appear that the total linear momentum is approximately conserved in totally inelastic collision Experiment 1? Is the percent error on liner momentum reasonable? Why? Why not? What can be improved?

Table 2. Kinetic energies in one-dimensional totally inelastic collisions with one moving and one stationary cart.

Exp. 1	$KE_i = m_1 v_{1,i}^2/2$ (J)	$KE_f = (m_1+m_2)v_f^2/2$ (J)	err_{KE} (%)
A			
B			
C			

Based on Table 2, does it appear that the total kinetic energy is approximately conserved in totally inelastic collision Experiment 1? Is the percent error on kinetic energy reasonable? Why? Why not? What can be improved?

PROCEDURE - ELASTIC COLLISIONS

Definition: If the kinetic energy is conserved during collision, then the collision is called *elastic*.

Experimental setup for elastic collisions

1. Make sure that adjusting the leveling screws on the track feet levels the track. Place a cart at each end of the track. Let the cart on the left be 'cart 1' and the cart on the right be 'cart 2'.
2. The carts have magnetic bumpers at one end. Turn the carts such that the magnetic bumpers face each other.
3. Make sure that the interface is connected to the computer, turn on the interface, and turn on the computer.
4. Two motion sensors are necessary for this experiment. One motion sensor is connected into digital channels 1 and 2 and the other into channels 3 and 4. The yellow plug of a sensor is always connected into the odd channel.
5. Run DataStudio on the computer and open the file called "ellastic_collision.ds". The file corresponds to the physical setup shown in Fig. 4 and is responsible for collecting and drawing the values of the velocity of both the left and right cart.

Experiment 2. Elastic collision with one cart moving and one stationary (see Figure 4)

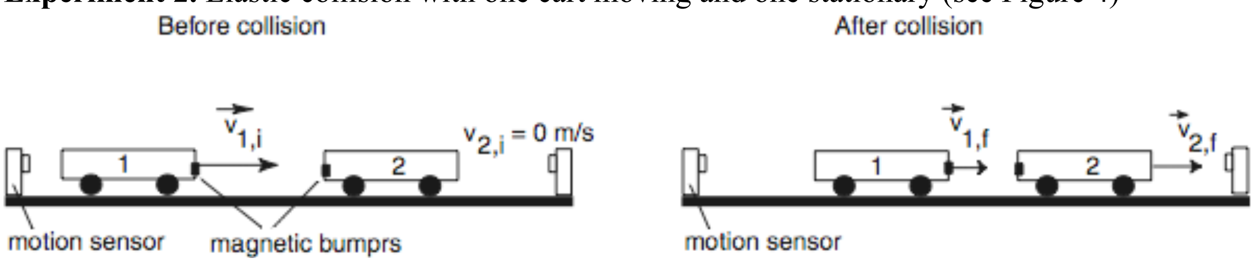

Figure 4. Elastic collision setup with one cart at rest and one in motion.

Prediction

Suppose two carts of masses m_1 and, respectively, m_2 collide elastically. Knowing that before collision the first cart was in motion and the second at rest (see Fig. 4), what would you expect the final velocity of each mass to be after collision? Explain your reasoning and show all your calculations.

Perform the following three experiments in order to check your prediction:

A. Equal mass carts.

Use the triple beam balance to measure the mass of each cart and record the values section A of Table 3. Record the velocities obtained from steps 1 and 2 below in section A of Table 3.

1. With cart 2 stationary, give cart 1 a quick push and collect data as it coasts forward (the positive direction is always considered away from motion detector). Stop recording soon after the two carts collide and before either cart hits a stopper. Practice until you obtain a graph of velocity versus time that has a distinct region of approximately constant velocity both before and after the collision of each of the carts.
2. When you are satisfied with your data, carefully draw and clearly label the velocity graph of each cart in the appropriate panel of Fig. 5.

B. Heavy moving cart hitting a lighter stationary cart.

Place a mass bar in the bed of the first cart such that the heavy cart hits the lighter cart at rest and repeat the steps 1 and 2 of Experiment 2A above. Repeat this section of the experiment with two and three bar weights on moving cart. Record the values in section B of Table 3. Carefully draw and clearly label the velocity graph of each cart in the appropriate panel of Fig. 5 Don't forget which cart is which!

C. Light moving cart hitting the heavier stationary cart.

Place a mass bar in the bed of the second (stationary) cart such that the lighter cart hits the heavier cart at rest and repeat steps 1 and 2 of Experiment 2A above. Repeat the experiment with two and three bar weights on the stationary cart. Record the values in section C of Table 3. Carefully draw and clearly label the velocity graph of each cart in the appropriate panel of Fig. 5 Do not forget which cart is which!

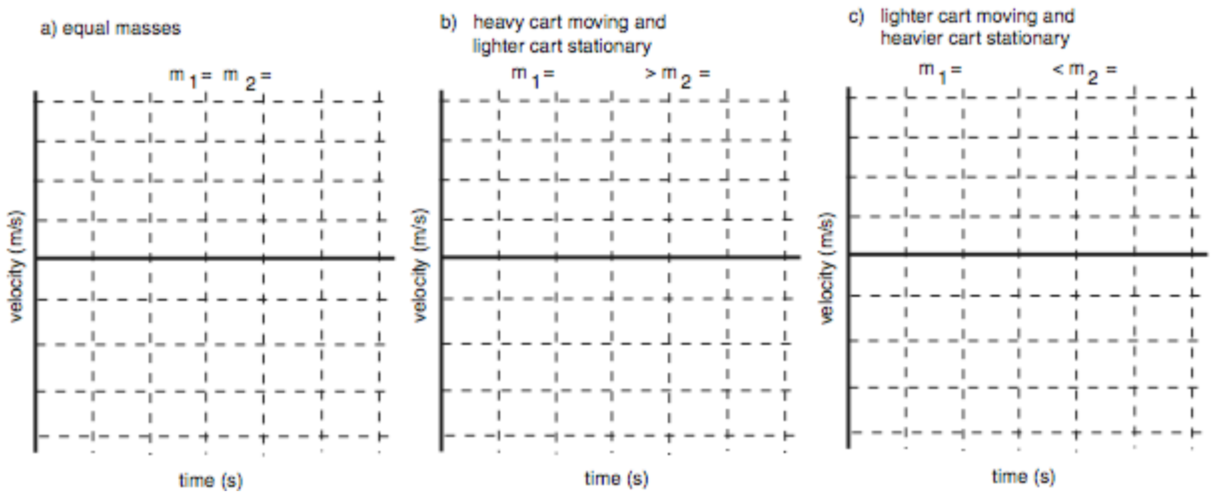

Figure 5. Sketch of the velocity versus time graph for one-dimensional elastic collision with (a) equal masses, (b) a heavy moving cart hitting a lighter stationary cart, and (c) a light moving cart hits a heavier stationary cart. Mark on the graph the region over which the collision occurs.

Table 3. Momentum in one-dimensional elastic collision with one cart moving and one stationary.

Exp. 2	m_1 (kg)	m_2 (kg)	Before collision	After collision		$p_i = m_1 v_{1,i}$ (kg m/s)	$p_f = m_1 v_{1,f} + m_2 v_{2,f}$ (kg m/s)	err_P (%)
			$v_{1,i}$ (m/s)	$v_{1,f}$ (m/s)	$v_{2,f}$ (m/s)			
A								
B								

Based on Table 3, does it appear that the total momentum is approximately conserved in elastic collision Experiment 2? Is the percent error on liner momentum reasonable? Why? Why not? What can be improved?

Table 4. Kinetic energies in one-dimensional elastic collision with one moving cart and one stationary cart.

Exp. 2	$KE_i = m_1 v_{1,i}^2/2$ (J)	$KE_f = m_1 v_{1,f}^2/2 + m_2 v_{2,f}^2/2$ (J)	err_{KE} (%)
A			
B			

Based on Table 4, does it appear that the total kinetic energy is approximately conserved in elastic collision Experiment 2? Is the percent error on kinetic energy reasonable? Why? Why not? What can be improved?

Experiment 3. Elastic collision with carts moving toward each other

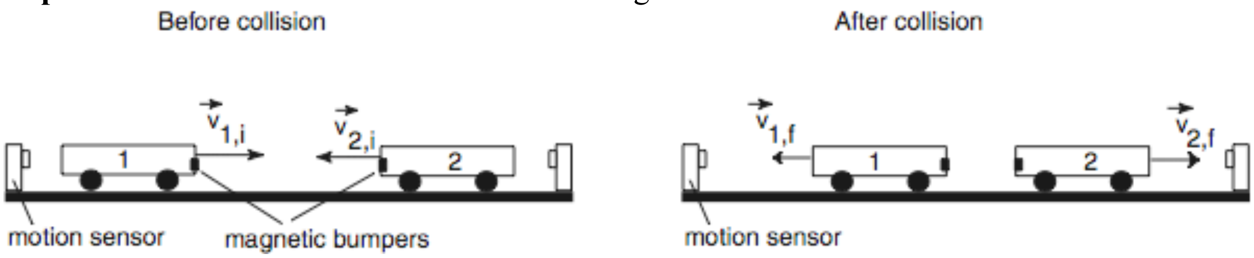

Figure 6. Elastic collision with both carts moving toward each other.

Prediction

Suppose two carts of masses m_1 and, respectively, m_2 collide elastically. Knowing that before collision the carts move toward each other (Fig. 6), what would you expect the final velocity of each mass to be after collision? Explain your reasoning and show all your calculations.

Perform the following two experiments to check your prediction:

A. Equal mass carts.

Use the triple beam balance to find the mass of each cart and record the values in section A of Table 6. Proceed to the next step without adding any bar weights on top of the carts. For the third experiment, clear all data recorded for the previous experiment or close the "ellastic_collision.ds" without saving your results and open it again. The file corresponds to the same physical setup as before and collects the values of the velocity of the left and right carts at the same time. Record the velocities obtained from steps 1 and 2 below in section A of Table 5.

1. Give both carts a quick push such that each coasts towards the other and they collide. Stop recording right after the two carts collide and before either cart hits a stopper. Practice until you obtain a graph of velocity versus time that has a distinct region of approximately constant velocity both before and after the collision for each of carts.
2. When you are satisfied with your data, carefully draw and clearly label the velocity graph for both carts in the appropriate panel of Fig. 7.

B. Carts with different masses moving toward each other.

Place a mass bar on top of the first cart such that the heavy cart hits the lighter cart moving toward each other and repeat steps 1 and 2 of Experiment 3A above. Repeat the experiment with two or more bar weights on carts. Record the values in section B of Table 5. Carefully draw and clearly label the velocity graph of each cart in the appropriate panel of Fig. 7. Do not forget which cart is which!

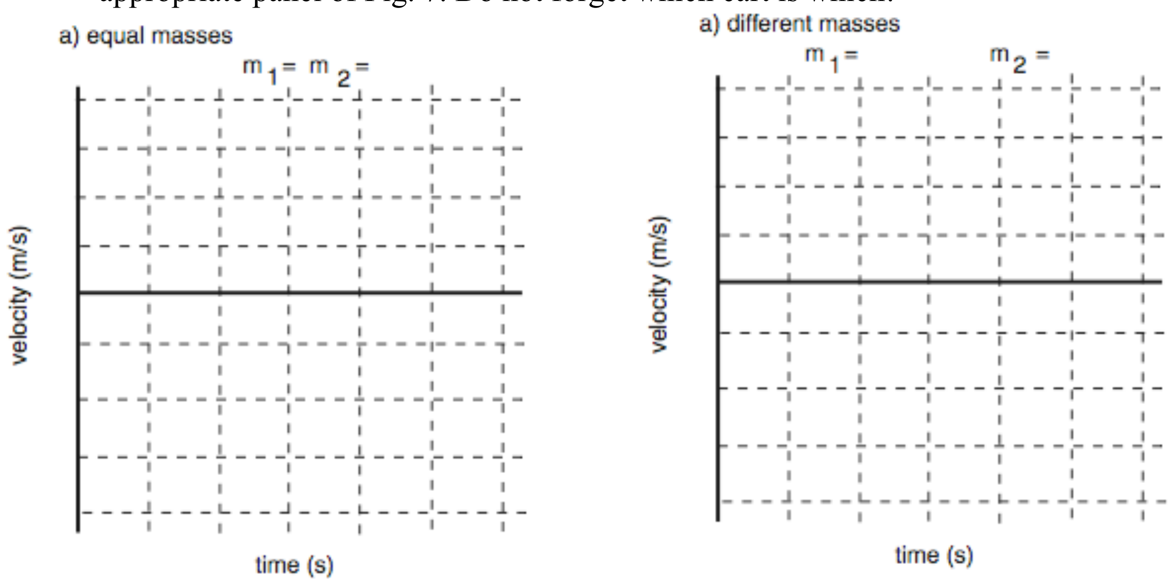

Figure 7. Sketch of the velocity versus time graph for elastic collision of carts moving toward each other and having (a) equal and (b) unequal masses. Mark on the graph the region over which the collision occurs.

Table 5. Momentum in one-dimensional elastic collision with the carts moving toward each other.

Exp. 3	m_1 (kg)	m_2 (kg)	Before collision		After collision		$p_i = m_1v_{1,i} + m_2v_{2,i}$ (kg m/s)	$p_f = m_1v_{1,f} + m_2v_{2,f}$ (kg m/s)	err_P (%)
			$v_{1,i}$ (m/s)	$v_{2,I}$ (m/s)	$v_{1,f}$ (m/s)	$v_{2,f}$ (m/s)			
A									
B									

Based on Table 5, does it appear that the total momentum is approximately conserved in elastic collision Experiment 3? Is the percent error on liner momentum reasonable? Why? Why not? What can be improved?

Table 6. Kinetic energies in one-dimensional elastic collision with the carts moving toward each other.

Exp. 3	$KE_i = m_1v_{1,i}^2/2 + m_2v_{2,i}^2/2$ (J)	$KE_f = m_1v_{1,f}^2/2 + m_2v_{2,f}^2/2$ (J)	err_{KE} (%)
A			
B			

Based on Table 6, does it appear that the total kinetic energy is approximately conserved in elastic collision Experiment 3? Is the percent error on kinetic energy reasonable? Why? Why not? What can be improve?